17. **Remark.** One version of Hyers's theorem states that if G is an Abelian group and f is a complex-valued function on G for which the function $(x, y) \mapsto f(xy) - f(x) - f(y)$ is bounded, then there exists a complex homomorphism a of G such that $f - a$ is bounded. Forti has given an example which shows that this theorem is not true if G is noncommutative. Now we prove the following theorem.

THEOREM. *Let G be a group which admits an invariant mean, that is, a positive, translation-invariant linear functional M, defined on the Banach space of all bounded complex-valued functions and normalized by $M(1) = 1$. Then, if f is a complex-valued function on G for which the function $(x, y) \to f(xy) - f(x) - f(y)$ is bounded, then there exists a complex homomorphism a of G such that $f - a$ is bounded.*

Proof. Let $a(y) = M_x[f(xy) - f(x)]$ for all y in G, where M_x indicates that M is applied with respect to the variable x. Then, by the properties of M, we have that

$$\begin{aligned} a(z) + a(y) &= M_x[f(xz) - f(x)] + M_x[f(xy) - f(x)] \\ &= M_x[f(xz) - f(x)] + M_x[f(xzy) - f(xz)] \\ &= M_x[f(xzy) - f(x)] = a(zy) \end{aligned}$$

and hence a is a homomorphism. On the other hand

$$|a(y) - f(y)| = |M_x[f(xy) - f(x)] - f(y)| = |M_x[f(xy) - f(x) - f(y)]|$$

which is bounded.

This observation is interesting because the only known examples for groups, which do not admit invariant means, are the groups containing a subgroup on two free generators. A celebrated conjecture in the theory of invariant means is that these are the only ones. On the other hand, in the example of Forti, the group was just the free group on two generators. Perhaps there may be a deeper connection between invariant means and Hyers' theorem.

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